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ΕΡΓΑΣΙΑ

Κ04. ΔΙΟΙΚΗΣΗ ΟΡΓΑΝΙΣΜΩΝ ΚΑΙ ΕΠΙΧΕΙΡΗΣΕΩΝ ΜΕΤΑΦΟΡΩΝ ΚΑΙ ΤΕΧΝΙΚΕΣ ΥΠΟΣΤΗΡΙΞΗΣ ΛΗΨΗΣ ΑΠΟΦΑΣΕΩΝ

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TRANSPORTATION RESEARCH PART B

Modeling Advanced Traveler Information Services: Static versus Dynamic paradigms

Hong K. Lo, W.Y. Szeto

Το παρόν αποτελεί μία περιληπτική περιγραφή του παραπάνω άρθρου, που τελεί υπό δημοσίευση στο περιοδικό TRANSPORTATION RESEARCH PART B. Αντικείμενο του άρθρου αποτελεί η μοντελοποίηση και διερεύνηση των επιπτώσεων της εισαγωγής των Σύγχρονων Συστημάτων Πληροφόρησης Οδηγού (ATIS - Advanced Traveler Information Services) στα δίκτυα.

Αν και στο παρελθόν έχουν πραγματοποιηθεί διάφορες σχετικές μελέτες, η πρωτοτυπία της συγκεκριμένης περίπτωσης έγκειται στο ότι ο βαθμός διείσδυσης των ATIS στο δίκτυο (αριθμός χρηστών που αξιοποιούν την υπηρεσία) δεν θεωρείται δεδομένος και σταθερός, αλλά μεταβάλλεται ανάλογα με τα οφέλη σε χρόνο διαδρομής που προσφέρει η υπηρεσία στους χρήστες κάθε συγκεκριμένου δρομολογίου.

Για τη διερεύνηση αξιοποιείται ένα δυναμικό μοντέλο προσομοίωσης της κυκλοφορίας, βασισμένο στο επονομαζόμενο Cell Transmission Model (CTM). Επιπλέον, λαμβάνει χώρα προσομοίωση και με ένα στατικό μοντέλο, που χρησιμοποιεί τη συνήθη συνάρτηση τύπου BPR. Τα δύο μοντέλα εφαρμόζονται στην περίπτωση ενός πρότυπου δικτύου, με σκοπό τη σύγκριση των αποτελεσμάτων από τις δύο θεωρήσεις, στατική και δυναμική.

Σημειώνεται ότι το πρωτότυπο υπό δημοσίευση άρθρο παρατίθεται αυτούσιο στο Παράρτημα που ακολουθεί.

1. Συντομογραφίες

Π-Π	Προέλευση - Προορισμός
ATIS	Advanced Traveler Information Services
BPR	Bureau of Public Roads
CTM	Cell Transmission Model
ISP	Information Service Provider
MP	Market Penetration
NCP	Non-linear Complementarity Problem
RT	Reduction in Time
SDUO	Stochastic Dynamic User Optimal
SO	System Optimum
SUE	Stochastic User Equilibrium
TMA	Traffic Management Agency
UB	User Benefit
UE	User Equilibrium
VIP	Variational Inequality Problem

2. Δυναμικό και στατικό μοντέλο - Οι παράμετροι των συστημάτων ATIS

Για τις ανάγκες της παρούσας διερεύνησης πραγματοποιούνται οι παρακάτω παραδοχές:

- Οι συνολικές ζητήσεις ανά ζεύγος Π-Π θεωρούνται δεδομένες και σταθερές
- Δεν υπάρχει κάποιο συμβάν στο δίκτυο, ήτοι οι χωρητικότητες (capacities) των συνδέσμων (links) είναι σταθερές

- Οι χρήστες δεν έχουν ιδεώδη πληροφόρηση για την κατάσταση του δικτύου, επομένως οι χρόνοι διαδρομής που αντιλαμβάνονται εμπεριέχουν ένα βαθμό μεταβλητότητας, σύμφωνα με την προσέγγιση SUE
- Η παρούσα διερεύνηση εστιάζεται στην ισορροπία του δικτύου (equilibrium) παρά στις αντιδράσεις των οδηγών σε συμβάντα

Για τη διερεύνηση θεωρείται ένα δίκτυο με διάφορα ζεύγη Π-Π και δύο κατηγορίες οδηγών, οι εφοδιασμένοι με υπηρεσία ATIS και αντίστοιχα οι μη. Το χρονικό πεδίο της μελέτης $[0,T]$ χωρίζεται σε M διαστήματα χρόνου δ, ώστε $T = M \cdot \delta$, και θεωρείται επαρκές για την εξυπηρέτηση όλης της ζήτησης. Το δίκτυο διαχωρίζεται σε κελιά, κατά την ανάλυση του μοντέλου CTM.

Σαν βασική αρχή της ανάλυσης θεωρείται ότι όλοι οι χρήστες, εφοδιασμένοι και μη με την υπηρεσία ATIS, ακολουθούν την κατάσταση SDUO[1].

Στη μοντελοποίηση υπεισέρχονται τρεις παράγοντες: (i) η βασική αρχή επιλογής διαδρομής, (ii) ο βαθμός διείσδυσης της υπηρεσίας ATIS στο σύνολο των χρηστών, και (iii) ο καθορισμός των χρόνων διαδρομής.

Όπως προαναφέρθηκε, η ***βασική αρχή επιλογής διαδρομής*** ακολουθεί την κατάσταση SDUO για αμφότερους τους εφοδιασμένους και μη χρήστες. Σύμφωνα με το Εκθετικό Μοντέλο (Logit), για καθεμία από τις δύο κατηγορίες χρηστών, και για κάθε δυνατή διαδρομή μεταξύ κάθε ζεύγους Π-Π, ισχύει η σχέση της μορφής:

[φόρτος διαδρομής] = [ποσοστό από τη ζήτηση Π-Π] × [ζήτηση ζεύγους Π-Π]

με τον όρο που εκφράζει το ποσοστό να ακολουθεί το Μοντέλο Logit. Ουσιαστικά, δηλαδή, η συνολική ζήτηση για ένα ζεύγος Π-Π μοιράζεται στις επιμέρους δυνατές διαδρομές που συνδέουν τα δύο σημεία, βάσει του ποσοστού που αναλογεί σε έκαστη από αυτές. Ο όρος που εκφράζει το ποσοστό κατανομής περιλαμβάνει το χρόνο κάθε διαδρομής του ζεύγους Π-Π για κάθε συγκεκριμένη χρονική στιγμή (λόγω της επιδιωκόμενης κατάστασης SDUO), καθώς και έναν ***παράγοντα ποιότητας πληροφόρησης για την κατάσταση του δικτύου***, για καθεμία από τις δύο κατηγορίες χρηστών (όπου και προφανώς οι εφοδιασμένοι χρήστες έχουν καλύτερη πληροφόρηση από τους μη). Αυτός είναι και ο τρόπος κατά τον οποίο η υπηρεσία ATIS υπεισέρχεται στην επιλογή διαδρομής.

Ο ***βαθμός διείσδυσης της υπηρεσίας ATIS στην αγορά*** υπεισέρχεται στη μοντελοποίηση κατά έναν ελαστικό τρόπο, κατανέμοντας τη συνολική ζήτηση ενός ζεύγους Π-Π σε ζήτηση από τους εφοδιασμένους χρήστες συν τη ζήτηση από τους αντίστοιχα μη. Η βασική σχέση που χρησιμοποιείται είναι:

[ζήτηση ζεύγους Π-Π εφοδιασμένων χρηστών] = $\Phi(C_N, B \cdot \varphi)$

όπου C_N είναι το πάγιο κόστος παροχής της υπηρεσίας (τιμή), B το κόστος χρόνου και φ το κέρδος σε χρόνο λόγω της αξιοποίησης της υπηρεσίας, με:

φ = [μέσος χρόνος στο ζεύγος για μη εφοδιασμένο όχημα] - [χρόνος για εφοδιασμένο]

όπου ο μέσος χρόνος για όλο το ζεύγος Π-Π προκύπτει ως το άθροισμα των χρόνων για κάθε επιμέρους διαδρομή μεταξύ των σημείων του ζεύγους. Έτσι, όσο αυξάνεται το κέρδος χρόνου λόγω της υπηρεσίας, τόσο αυξάνεται η ζήτηση από τα εφοδιασμένα οχήματα (δηλαδή έχουν εφοδιαστεί αντίστοιχα περισσότερα οχήματα με την υπηρεσία ATIS), δηλαδή ο βαθμός διείσδυσης της υπηρεσίας θεωρείται ότι εξαρτάται από τα άμεσα οφέλη για τους χρήστες σε κάθε ζεύγος Π-Π.

Ο ***καθορισμός των χρόνων διαδρομής*** πραγματοποιείται κάτω από δύο διαφορετικές οπτικές, με ένα δυναμικό και ένα στατικό μοντέλο αντίστοιχα, όπως τονίστηκε και παρα-

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[1] Σε διάφορες άλλες σχετικές εργασίες η αντιμετώπιση αυτή μπορεί να ποικίλλει, π.χ. με τους εφοδιασμένους οδηγούς να ακολουθούν SO και τους μη εφοδιασμένους UE, ή UE και SUE αντίστοιχα.

πάνω. Σαν δυναμικό μοντέλο αξιοποιείται το CTM, με το χρόνο κάθε επιμέρους διαδρομής για κάθε ζεύγος Π-Π να αποτελεί συνάρτηση όλων των φόρτων στο δίκτυο:

[χρόνος συγκεκριμένης διαδρομής] = $\Phi(f_1, f_2)$

όπου f_1 και f_2 είναι τα διανύσματα των φόρτων όλων των δυνατών διαδρομών στο δίκτυο, για εφοδιασμένα και μη, αντίστοιχα, οχήματα. Σαν στατικό μοντέλο χρησιμοποιείται το γνωστό μοντέλο με τη συνάρτηση τύπου BPR. Είναι δεδομένο ότι ένα δυναμικό μοντέλο προσομοιώνει με πολύ μεγαλύτερη ακρίβεια ένα δίκτυο. Το ζήτημα, επομένως, είναι κατά πόσο η απλότητα του στατικού μοντέλου είναι αρκετή για να αναπαραστήσει τη λειτουργία ενός συστήματος ATIS στο δίκτυο.

Τέλος, για την ***επίλυση του δικτύου*** οι τρεις παραπάνω παράγοντες (βασική αρχή επιλογής διαδρομής, βαθμός διείσδυσης υπηρεσίας, καθορισμός χρόνων διαδρομής με στατικό ή δυναμικό μοντέλο) εισάγονται σε μία διαδικασία τύπου NCP (Non-linear Complementarity Problem) ή αντίστοιχη ισοδύναμη τύπου VIP (Variational Inequality Problem). Το ζητούμενο αποτέλεσμα είναι οι φόρτοι σε όλες τις διαδρομές του δικτύου, και επομένως και οι αντίστοιχοι χρόνοι.

3. Δείκτες απόδοσης

Για τη διευκόλυνση της σύγκρισης μεταξύ δυναμικού και στατικού μοντέλου, ορίζονται ορισμένοι δείκτες απόδοσης της λειτουργίας του δικτύου. Η απόδοση μπορεί να θεωρηθεί από τη σκοπιά τριών διαφορετικών εμπλεκομένων στη λειτουργία του δικτύου, ήτοι των ***χρηστών*** (users), του ***παροχέα των υπηρεσιών ATIS*** (ISP - Information Service Provider), καθώς και του ***διαχειριστή του δικτύου*** (TMA - Traffic Management Agency).

Όσον αφορά το ***συνολικό κέρδος των χρηστών του δικτύου***, ο δείκτης απόδοσης έχει τη μορφή:

$$\mathrm{UB} = \frac{\sum_{\text{όλα τα Π-Π}} \left(\left[\text{ζήτηση ζεύγους Π - Π εφοδιασμένων χρηστών} \right] \times \left[\text{κέρδος χρήστη ζεύγους Π - Π} \right] \right)}{\sum_{\text{όλα τα Π}-\text{Π}} \left[\text{ζήτηση ζεύγους Π - Π εφοδιασμένων χρηστών} \right]}$$

όπου το κέρδος χρήστη ζεύγους Π-Π ορίζεται ως το κερδισμένο κόστος από την απομείωση του χρόνου της διαδρομής μείον το πάγιο κόστος (τιμή) παροχής της υπηρεσίας.

Ο δείκτης απόδοσης από τη σκοπιά του ***παροχέα των υπηρεσιών ATIS*** είναι το κέρδος του P:

P = $Q \cdot C_N$ - [κόστος επεξεργασίας] - [κόστος εξυπηρέτησης Q εφοδιασμένων χρηστών]

όπου Q το σύνολο της ζήτησης στο δίκτυο από εφοδιασμένους χρήστες και C_N η τιμή της υπηρεσίας.

Ο ***διαχειριστής του δικτύου*** επιθυμεί την ελαχιστοποίηση του συνολικού χρόνου διαδρομής σε όλο το δίκτυο, οπότε ο σχετικός δείκτης απόδοσης είναι το ποσοστό μείωσης RT του εν λόγω συνολικού χρόνου μετά την εισαγωγή της υπηρεσίας ATIS.

Τέλος, ένας ακόμη δείκτης απόδοσης θα μπορούσε να θεωρηθεί και ο ***βαθμός διείσδυσης της υπηρεσίας ATIS*** στο σύνολο των χρηστών, MP, που ορίζεται απλά ως το ποσοστό εφοδιασμένων χρηστών σε σχέση με το σύνολο των χρηστών του δικτύου.

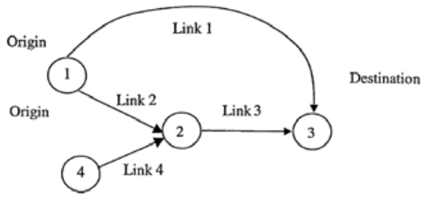
4. Μεθοδολογία σύγκρισης

Για την πραγματοποίηση της σύγκρισης της προσομοίωσης, το δυναμικό και το στατικό μοντέλο εφαρμόζονται στην περίπτωση ενός προτύπου δικτύου, και για το ίδιο σενάριο κατάστασης. Η μεθοδολογία ακολουθεί τα εξής βήματα:

(i) Υπόθεση ενός ζεύγους (θ_1, C_N) (ποιότητα και τιμή υπηρεσίας αντίστοιχα)
(ii) Επίλυση στατικού μοντέλου για το επιλεγμένο ζεύγος του βήματος (i)
(iii) Εκτίμηση δεικτών απόδοσης Παραγράφου 3
(iv) Επανάληψη των βημάτων (ii), (iii) για διάφορα άλλα ζεύγη (θ_1, C_N)
(v) Παραγωγή διαγραμμάτων των δεικτών απόδοσης σε άξονες θ_1 - C_N
(vi) Επανάληψη των βημάτων (i) έως (v) για το δυναμικό μοντέλο

5. Εφαρμογή

Το δυναμικό και το στατικό μοντέλο εφαρμόστηκαν στην περίπτωση ενός πρότυπου δικτύου, και για δύο διαφορετικά σενάρια λειτουργίας, για χαμηλή και για υψηλή ζήτηση κυκλοφορίας αντίστοιχα. Οι διάφορες παράμετροι λειτουργίας του δικτύου και οι συντελεστές των μοντέλων επελέγησαν ώστε να είναι παρόμοιες για τα δύο είδη μοντέλων. Το δίκτυο φαίνεται στο **Σχήμα 1**.

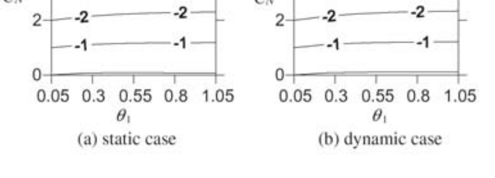


Σχήμα 1: Το πρότυπο δίκτυο.

Εφαρμόζοντας τη μεθοδολογία για την περίπτωση του σεναρίου ***υψηλής ζήτησης***, προκύπτουν τα παρακάτω αποτελέσματα:

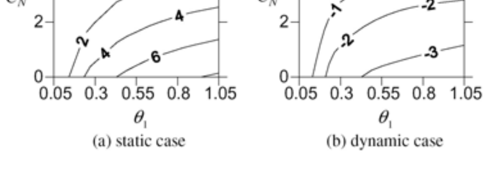
- Χρόνοι διαδρομής, φόρτοι και κέρδος χρόνου (άρα και βαθμός διείσδυσης) ποικίλλουν ανάλογα με το χρόνο αναχώρησης, κάτι που δεν μπορεί να προσομοιωθεί με το στατικό μοντέλο
- Τα διαγράμματα των δεικτών απόδοσης του συνολικού κέρδους των χρηστών (UB), του κέρδους του παροχέα υπηρεσιών (P) και του βαθμού διείσδυσης της υπηρεσίας (MP) προκύπτουν παρόμοια για τα δύο μοντέλα
- Το διάγραμμα του δείκτη απόδοσης συνολικής μείωσης χρόνου διαδρομής στο δίκτυο (RT) προκύπτει εντελώς διαφορετικό για τα δύο μοντέλα

Ασφαλώς τα αποτελέσματα αφορούν το συγκεκριμένο δίκτυο, με τις συγκεκριμένες παραμέτρους που προσομοιώθηκαν. Ο λόγος για τον οποίο οι δείκτες UB, P και MP προέκυψαν παρόμοιοι για το δυναμικό και στατικό μοντέλο έγκειται στο ότι αμφότερα τα μοντέλα



(a) static case
(b) dynamic case

Σχήμα 2: Δείκτης UB για τα δύο είδη μοντέλων.



(a) static case
(b) dynamic case

Σχήμα 3: Δείκτης RT για τα δύο είδη μοντέλων.

προέβλεψαν εν γένει χαμηλά οφέλη χρόνου από τη χρήση της υπηρεσίας. Έτσι, το κέρδος των χρηστών UB καθορίζεται κατά κύριο λόγο από την τιμή της υπηρεσίας C_N, και όχι από την ακριβή διαφοροποίηση του οφέλους σε χρόνο από τα δύο μοντέλα (**Σχήμα 2**). Ταυτόχρονα, το ίδιο ισχύει και για τα P και MP, που μεταβάλλονται μονοτονικά με το UB.

Ο δε λόγος για τον οποίο τα δύο μοντέλα έδωσαν τελείως αντιδιαμετρικά αποτελέσματα για το δείκτη συνολικής μείωσης χρόνου διαδρομής RT στο δίκτυο από τη χρήση της υπηρεσίας (**Σχήμα 3**), έγκειται στον τρόπο προσομοίωσης της λειτουργίας του δικτύου από τα δύο μοντέλα, δυναμικό και στατικό. Φαίνεται δε ότι η διαφορά στην εικόνα της κατάστασης του δικτύου από τα δύο μοντέλα επηρεάζει την ίδια τη διαδικασία λήψης απόφασης δρομολογίου από τους χρήστες που είναι εφοδιασμένοι με την υπηρεσία ATIS, για την επιδίωξη της κατάστασης SDUO. Ασφαλώς είναι δεδομένο ότι το δυναμικό μοντέλο είναι πολύ πιο ακριβές στην αναπαράσταση της λειτουργίας του δικτύου.

Στην περίπτωση του σεναρίου ***χαμηλής ζήτησης***, η ανυπαρξία συμφόρησης στο δίκτυο οδηγεί σε παρόμοια αποτελέσματα απεικόνισης της κατάστασης σε αυτό από τα δύο μοντέλα, οπότε παρόμοια είναι πλέον και όλα τα διαγράμματα των δεικτών απόδοσης.

6. Συμπεράσματα

Αν και η διερεύνηση που προηγήθηκε αφορά στο συγκεκριμένο δίκτυο που προσομοιώθηκε, από τα αποτελέσματά της μπορεί να εξαχθεί το συμπέρασμα ότι αν και κάποια αποτελέσματα από την εφαρμογή των δύο ειδών μοντέλων είναι παρόμοια, δεν αποκλείεται κάποια άλλα να είναι τελείως διαφορετικά, στην περίπτωση που επιχειρείται να προσομοιωθεί η ύπαρξη υπηρεσίας ATIS. Το κύριο εξαγόμενο είναι ότι σε αυτήν την περίπτωση ένα απλό στατικό μοντέλο δεν είναι αρκετό για να αναπαραστήσει επαρκώς ένα δυναμικό δίκτυο, προπαντός σε περίπτωση συμφόρησης.

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Modeling advanced traveler information services: static versus dynamic paradigms

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Abstract

This paper develops a cell-based variational inequality formulation of the dynamic traffic assignment (DTA) problem to evaluate the impact of advanced traveler information systems (ATIS) services. It considers two classes of drivers: those with ATIS and those without. Both classes are modeled to follow the stochastic dynamic user optimal conditions, with the equipped drivers having a lower perception variation of the network travel time due to the availability of better information. The model represents traffic dynamics according to the cell-transmission model, including such physical effects as queue spillback, shockwaves, etc. One objective of this study is to compare and contrast the static versus the dynamic modeling paradigms for this problem. The numerical study indicates that some aspects of the results from these two paradigms could be diametrical. The discrepancy is mainly attributed to the fundamental characteristic of a model—how traffic is represented. It appears that simplifications from the physical queue representation are inadequate in producing correct results, especially for already congested networks wherein junction blockage is common.

1. Introduction

Many studies have been undertaken to model the benefits of advanced traveler information systems (ATIS). The crux of these studies focuses on the route choice behaviors of the equipped and unequipped vehicles, including user-equilibrium (UE), system-optimum (SO), and stochastic user-equilibrium (SUE). Among them, Kanafani and Al-Deek (1991) estimated the benefits of

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ATIS by comparing the UE versus SO system times. Harker (1988), Van Vuren et al. (1989), and Bennett (1993) considered that the equipped vehicles follow the SO routes while the unequipped vehicles follow the UE routes. Other efforts, including Koutsopoulos and Lotan (1990), modeled the equipped vehicles as following the UE routes whereas the unequipped vehicles as following the SUE routes. This approach is behaviorally more appealing, as each type of vehicles is modeled to strive for better travel time subject to their perception variations of the network travel times.

All of the above studies, however, assume a fixed market penetration of ATIS services, regardless of individuals' trip lengths or potential travel time savings. In reality, people subscribe to ATIS services because of their benefits. Therefore, the demand for ATIS services should be an elastic function of benefits, as considered in Yang (1998). The end result is that the market penetrations of ATIS services are different for different origin–destination (OD) pairs. In Lo and Szeto (2001, 2002a), we further considered the equipped and unequipped travelers as following two SUE traffic patterns, with the equipped ones having a lower perception variation of the network travel time. The extent of this travel time perception variation of the equipped travelers is considered as a measure of the information quality of the ATIS services to be provided.

Most previous studies employ static models for an essentially dynamic problem. Static models do not permit the study of changes in travelers' departure time, dynamic queuing locations and duration, non-recurrent congestion, etc., which may bias the analysis. More recently, some studies began to analyze the problem with a dynamic paradigm (e.g. Al-Deek and Kanafani, 1993; Al-Deek et al., 1998; Arnott et al., 1991; Ben-Akiva et al., 1991; Chen and Mahmassani, 1991; Emmerink et al., 1995a,b; Hall, 1993; Mahmassani and Liu, 1997). As compared with static models, dynamic models can capture the problem more realistically, at the expense of increases in complexity and solution effort. The question is whether this endeavor is worthwhile. In particular, it is useful to find out whether the static and dynamic paradigms produce comparable results or indicate similar trends.

To answer this question, this study extends Lo and Szeto's (2002a) static ATIS model to a dynamic one by taking advantage of the dynamic route choice platform developed in Lo and Szeto (2002b). Similar to the static model, this new dynamic model considers the impacts of ATIS services from three perspectives: users (or service subscribers), Information Service Provider (ISP), and the Traffic Management Agency (TMA). Users are concerned with the benefits of the services primarily in terms of travel-time savings. ISP attends to the profitability of the services; whereas TMA mainly looks after the changes in overall network congestion.

The proposed dynamic model is formulated with the variational inequality (VI) approach as extended from Lo and Szeto (2002b). As an analytical formulation, it has well-defined route choice properties. Both the equipped and the unequipped vehicles are modeled to follow the stochastic dynamic user optimal (SDUO) principle (Ran and Boyce, 1996). As in Lo and Szeto (2002b), this dynamic model encapsulates a network version of the cell-transmission model (CTM) (Daganzo, 1994, 1995) so as to capture the physical effects of traffic dynamics, such as shockwaves, queue spillback, and dynamic traffic interactions across links. Moreover, we extend the consideration of ATIS services adoption to be dependent on travelers' departure times, in addition to being OD specific.

We set up two scenarios to compare the static and dynamic models. The main objective is to highlight whether these two modeling approaches produce similar results. The results, albeit based on a simple network, are revealing. They show that these two models give rather different benefit predictions. More importantly, the two models arrive at diametrically opposite conclusions in some cases: the static model predicts that ATIS would improve network congestion whereas the dynamic model indicates the reverse. A more detailed analysis shows that these differences can be attributed to how traffic is modeled. Specifically, whether physical queues are modeled properly or not affects the results in a substantial manner. It is anticipated that this discrepancy, as a fundamental characteristic, will arise in other studies that involve traffic dynamics modeling. Accordingly, advancing the modeling paradigm to one that considers dynamic queuing properly is essential for accuracy.

The outline of this paper is as follows. Section 2 develops the dynamic mixed-equilibrium traffic assignment model vis-à-vis the static model described in Lo and Szeto (2002a). Section 3 formulates the benefit of each party and the aggregated market penetration of ATIS services. Section 4 depicts the comparison methodology and the descent solution method adopted. Section 5 contains the numerical studies. Finally, Section 6 provides some concluding remarks.

2. Static and dynamic mixed-equilibrium formulations

In this section, we depict the dynamic mixed-equilibrium traffic assignment model in reference to the static model described in Lo and Szeto (2002a). By formulating the mixed-equilibrium problem through the non-linear complementarity problem (NCP) approach, the static and dynamic models share a similar structure in terms of representing the route choice principle and the ways to model ATIS market penetration. Nevertheless, the fundamental difference between the static and dynamic models is two-folded. First, the dynamic model requires an explicit representation of the time dimension within the formulation, which is not needed in the static model. Second, more importantly, in the dynamic model, the consistency between traffic flow, traffic dynamics, and travel time must be maintained through the underlying traffic flow model, whereas the static model simplifies this aspect by the use of a link performance function.

In the formulations of this study, the following assumptions are made:

- Total OD demands are considered as given and fixed.
- There is no incident on the network, therefore link capacities are fixed.
- Travelers do not have perfect information about the network condition; network travel times are subject to perception variations as modeled with the stochastic user-equilibrium approach.
- This study focuses on the equilibrium traffic patterns of the network rather than travelers' responses or reactions to incidents.

We consider a general transportation network with multiple OD flows and two classes of drivers: those with ATIS services (Class 1) and those without (Class 2). The study horizon $[0, T]$ is discretized into M intervals of length δ such that $T = M \cdot \delta$, which is assumed to be long enough to allow all traffic to clear the network. The network is divided into cells, following the principles depicted in Daganzo (1994, 1995).

The following notations are adopted throughout this paper:

(a) Indices

rs	OD pair
p	route between OD pair rs
t	index for departure time, $t \in \{1, 2, \ldots, N\}$, where $N \leqslant M$
i	driver type, $i = 1$ for subscribed drivers, $i = 2$ for non-subscribed drivers
a	link

(b) Variables to be determined

$f_{p,i}^{rs}(t)$	route flow on p between OD pair rs departing at time t for Class i travelers [veh/time step]
$\mathbf{f}_i$	column vector of $(f_{p,i}^{rs}(t), \forall rs, p, t)$ with dimension n_i for Class i drivers

(c) Parameters given

θ_i	parameter representing travel time variation of Class i travelers, interpreted as the information quality available to them [min^{-1}]
C_N	net or out-of-pocket service charge, which is a decision variable in designing ATIS services [\$]
B	value of time [\$/min]
ψ	general parameter in market penetration function capturing other benefits such as convenience of having the device, etc. For simplicity, this study assumes $\psi = 0$ [\$]
δ_a^p	link-path incidence indicator, $\delta_a^p = 1$ if a is on p, $\delta_a^p = 0$ otherwise

(d) Functions of the equilibrium route flows of equipped and unequipped travelers

$w_{p,i}^{rs}(t)$	proportion of the Class i travelers taking route p at departure time t
$q_i^{rs}(t)$	demand of Class i travelers between OD pair rs at time t [veh/time step]
$\tilde{q}^{rs}(t)$	total demand between OD pair rs at time t [veh/time step]
$\eta_p^{rs}(t)$	average route travel time on route p between rs for flows departing at time t [min]
$\phi^{rs}(t)$	travel time saving of equipped travelers on rs departing at time t [min]
y_a	flow on link a [vph]
t_a	travel time on link a [min]

The mixed-equilibrium model consists of three components: (i) route choice principle, (ii) modeling ATIS market penetration, and (iii) route travel time determination.

2.1. Route choice principle

The route choice behaviors of the two types of travelers, the ones equipped and unequipped with ATIS, are characterized by two sets of the SDUO conditions. The SDUO conditions state that for each OD pair at each instant of time, the actual travel times perceived by travelers departing at the same time are equal and minimal (Ran and Boyce, 1996). By adopting the logit model, mathematically these SDUO conditions for the equipped and unequipped travelers can be expressed as:

$$f_{p,i}^{rs}(t) = w_{p,i}^{rs}(t) \cdot q_i^{rs}(t) \quad \text{or} \quad f_{p,i}^{rs}(t) - w_{p,i}^{rs}(t) \cdot q_i^{rs}(t) = 0, \quad \forall rs, p, t, i, \tag{1}$$

where

$$w_{p,i}^{rs}(t) = \frac{\exp\left(-\theta_i \cdot \eta_p^{rs}(t)\right)}{\sum_k \exp\left(-\theta_i \cdot \eta_k^{rs}(t)\right)},$$

where $f_{p,i}^{rs}(t)$ and $w_{p,i}^{rs}(t)$ are respectively the flow and proportion of class-i travelers on route p between OD pair rs departing at time t; $q_i^{rs}(t)$ is the demand of class-i travelers between OD pair rs at time t; $\eta_p^{rs}(t)$ denotes the average route travel time on p between OD pair rs for travelers departing at t. The parameters θ_1 and θ_2, respectively, represent the perception variations of the equipped and unequipped travelers. According to the logit model, θ_i is inversely proportional to the standard deviation of the perceived travel time σ_i, expressed as: $\sigma_i = \frac{\pi}{\sqrt{6}\theta_i}$. A higher θ_1 (or θ_2) means smaller travel time perception variations and, hence, better information quality. In general, the equipped vehicles have a higher θ_1 than the unequipped ones (i.e. $\theta_1 > \theta_2$). The value of θ_2, on the other hand, is related to how familiar are travelers of the network conditions without ATIS services. This is a function of the network topology and composition of travelers: commuters versus visitors. One would expect that a simpler network with fewer route choice alternatives and a larger proportion of commuter traffic would have a higher θ_2. Nevertheless, both of these parameters can be calibrated to the quality of information available (Lo and Szeto, 2002a). In the limiting case when θ_1 or θ_2 approaches infinity, the corresponding route flow pattern approaches that as modeled by the deterministic user optimal approach, in which travelers are assumed to have perfect information about the network status.

To obtain the static formulation, one only needs to drop the time index of the variables in (1). The above SDUO conditions are then reduced to the SUE conditions. Moreover, in a similar fashion, as θ_i approaches infinity, the corresponding route flow pattern of the static formulation approaches that of the deterministic user-equilibrium conditions.

2.2. Modeling ATIS market penetration

The market penetration of ATIS services is modeled in an elastic manner, partitioning the total demand $\tilde{q}^{rs}(t)$ into $q_1^{rs}(t)$ (those equipped with ATIS) and $q_2^{rs}(t)$ (those unequipped) for each OD pair rs and departure time t. The elastic market penetration function is defined as:

$$q_1^{rs}(t) = \frac{\tilde{q}^{rs}(t)}{1 + \exp\left(C_N - B \cdot \phi^{rs}(t) - \psi\right)}, \quad \forall rs, t, \tag{2}$$

where C_N denotes out-of-pocket service charge; B represents the value of time; ψ is a general parameter in market penetration function capturing other benefits such as convenience of having the device, etc. For simplicity, this study assumes $\psi = 0$. The variable $\phi^{rs}(t)$ represents the travel time saving of the equipped travelers on OD pair rs, defined as:

$$\phi^{rs}(t) = \left(\sum_p w^{rs}_{p,2}(t) \cdot \eta^{rs}_p(t)\right) - \left(\sum_p w^{rs}_{p,1}(t) \cdot \eta^{rs}_p(t)\right), \quad \forall rs, t. \tag{3}$$

The first (second) term on the right hand side of (3) represents the average travel time of the unequipped (equipped) vehicles on OD pair *rs* at time *t*, with each a function of the route flows $\mathbf{f}_1$ and $\mathbf{f}_2$. Thus, the demand for ATIS services $q^{rs}_1(t)$ is also a function of $\mathbf{f}_1$ and $\mathbf{f}_2$. As for the static formulation, one obtains the corresponding relationships by dropping time index in the variables of (2) and (3).

With the total travel demand between OD pair *rs*, $\tilde{q}^{rs}(t)$, fixed, one can define the unequipped travelers between the same OD pair to be:

$$q^{rs}_2(t) = \tilde{q}^{rs}(t) - q^{rs}_1(t), \quad \forall rs, t. \tag{4}$$

In terms of route choice principles and the ways to model ATIS market penetration, the static and dynamic models share the same structure. The main difference is that the dynamic model explicitly tracks the time index in the flow and travel time variables.

2.3. Route travel time determination

The two modeling paradigms differ fundamentally in their ways of modeling traffic and travel times. In the dynamic approach, we make use of a dynamic traffic flow model to produce a unique mapping from route flows to route travel times. In this study, we choose the CTM (Daganzo, 1994, 1995) as the underlying traffic flow model to reproduce accurate traffic dynamics. The details of encapsulating CTM for dynamic traffic assignment (DTA) can be found in Lo (1999) and Lo and Szeto (2002b,c). For the sake of completeness, the basic principles of CTM and the approach of determining actual route travel times are given in Appendix A. To sum up, for any given route flow vectors, $\mathbf{f}_1$ and $\mathbf{f}_2$, the travel time of each route, $\eta^{rs}_p(t)$, can be determined through CTM and the route travel time extraction procedure depicted in Lo and Szeto (2002b). For the sake of simplicity, we express route travel time $\eta^{rs}_p(t)$ as a function of route flows $\mathbf{f}_1$ and $\mathbf{f}_2$:

$$\eta^{rs}_p(t) = \Phi(\mathbf{f}_1, \mathbf{f}_2), \quad \forall rs, p, t, \tag{5}$$

where $\Phi(\cdot)$ represents the unique mapping between route travel times and route flows.

In the static model, we adopt the Bureau of Public Roads (BPR) function to model link travel times, which are then used for calculating route travel times. Specifically, given route flows $f^{rs}_{p,1}$ and $f^{rs}_{p,2}$, link flow y_a and travel time t_a for link *a* can be determined by the following respectively:

$$y_a = \sum_i \sum_p f^{rs}_{p,i} \cdot \delta^p_a, \quad \forall a, \tag{5a}$$

$$t_a = t^0_a \left[1 + \alpha \left(\frac{y_a}{c_a}\right)^{\beta}\right], \tag{5b}$$

where δ^p_a is the link-path incidence indicator—$\delta^p_a = 1$ if *a* is on *p*; $\delta^p_a = 0$ otherwise. t^0_a is the link's free flow travel time; c_a is its practical capacity; α and β are parameters. Eq. (5a) sums route flows of both traveler classes to form link flows. Eq. (5b) is the BPR function. Once link travel time t_a is known, route travel time η^{rs}_p can be found by:

$$\eta_p^{rs} = \sum_a t_a \cdot \delta_a^p, \quad \forall rs, p. \tag{5c}$$

One remark is that the BPR function only considers average traffic loads without knowledge of time dependent inflows, exit flows, or physical queue blockage. The BPR function, thus, provides a crude approximation of travel times, as compared with the dynamic approach that takes into account detailed traffic dynamics. Indeed, the key question is whether the simplicity offered by the static model and the BPR function is sufficiently refined to model the mixed-equilibrium problem.

2.4. Non-linear complementarity problem representation

Finally, we put the three components together through the NCP approach, including the route choice principle expressed in (1), the ATIS market penetration modeling expressed in (2)–(4), and the route travel time determination expressed in (5). Multiplying $f_{p,i}^{rs}(t)$ to the SDUO conditions (1), and adding the non-negativity conditions, we obtain the following non-linear complementarity conditions for the mixed-equilibrium problem:

$$\begin{aligned} &f_{p,i}^{rs}(t)\left(f_{p,i}^{rs}(t) - w_{p,i}^{rs}(t)\cdot q_i^{rs}(t)\right) = 0, \quad \forall rs, p, t, i, \\ &f_{p,i}^{rs}(t) \geqslant 0, \quad \forall rs, p, t, i, \\ &f_{p,i}^{rs}(t) - w_{p,i}^{rs}(t)\cdot q_i^{rs}(t) \geqslant 0, \quad \forall rs, p, t, i. \end{aligned} \tag{6}$$

According to (6), if $f_{p,i}^{rs}(t) > 0$, then (1) must be satisfied, or $f_{p,i}^{rs}(t)$ is apportioned according to the logit split expression. If $f_{p,i}^{rs}(t) = 0$, the term $(f_{p,i}^{rs}(t) - w_{p,i}^{rs}(t)\cdot q_i^{rs}(t))$ can take on any value. However, this will not happen as the SDUO conditions (1) assign a positive flow to each of the routes. Finally, the constraint $f_{p,i}^{rs}(t) - w_{p,i}^{rs}(t)\cdot q_i^{rs}(t) \geqslant 0$ is added for mathematical completeness, which is always satisfied at the SDUO solutions.

Defining

$$\mathbf{x} = \left[\mathbf{f}_1^{\mathrm{T}}, \mathbf{f}_2^{\mathrm{T}}\right]^{\mathrm{T}} \in R_+^{n_1+n_2}, \tag{7}$$

$$\mathbf{F}(\mathbf{x}) = \left[f_{p,i}^{rs}(t) - w_{p,i}^{rs}(t)\cdot q_i^{rs}(t)\right] \in R_+^{n_1+n_2}, \tag{8}$$

and substituting (2)–(5) into (8), the non-linear complementarity conditions (6) can be expressed as:

$$\text{Finding } \mathbf{x}^* \geqslant \mathbf{0} \text{ such that} : \mathbf{x}^{*\mathrm{T}} \cdot \mathbf{F}(\mathbf{x}^*) = \mathbf{0}, \quad \mathbf{F}(\mathbf{x}^*) \geqslant 0. \tag{9}$$

One thing to note is that as route travel time $\eta_p^{rs}(t)$ is a function of $\mathbf{f}_1$ and $\mathbf{f}_2$, according to (1)–(4), $w_{p,i}^{rs}(t)$ and $q_i^{rs}(t)$ can be expressed as functions of $\mathbf{f}_1$ and $\mathbf{f}_2$ as well. Therefore, the non-linear complementarity conditions (6) can be expressed by $\mathbf{f}_1$ and $\mathbf{f}_2$ or $\mathbf{x}$ alone.

The NCP (9) is depicted for exposition clarity. To take advantage of recent algorithms developed for variational inequality problem (VIP), we transform the NCP (9) to an equivalent VIP, stated as:

$$\text{Find } \mathbf{x}^* \text{ such that} : (\mathbf{x} - \mathbf{x}^*)^{\mathrm{T}}\mathbf{F}(\mathbf{x}^*) \geqslant 0, \quad \forall \mathbf{x} \in R_+^{n_1+n_2}, \tag{10}$$

where $\mathbf{x}$ and $\mathbf{F}(\mathbf{x})$ are as defined in (7) and (8), respectively. This transformation can be readily accomplished with the same mapping $\mathbf{F}(\mathbf{x})$.

Proposition 1. *The complementarity problem* (9) *is equivalent to the VIP* (10).

Proof. See Proposition 1.4 in Nagurney (1993). □

3. Performance measures

For ease of comparison, we put the performance measures of the static and dynamic models side by side. The provision of ATIS services often involves three parties: users, Information Service Provider (ISP) and Traffic Management Agency (TMA) (Lo and Szeto, 2002a). Users are concerned with net user benefit (UB), [1] defined as:

$$\text{Static model}: \quad \mathrm{UB} = \frac{\sum_{rs} q_1^{rs} \cdot \mathrm{ub}^{rs}}{\sum_{rs} q_1^{rs}}; \tag{11}$$

$$\text{Dynamic model}: \quad \mathrm{UB} = \frac{\sum_{rs} \sum_t q_1^{rs}(t) \cdot \mathrm{ub}^{rs}}{\sum_{rs} \sum_t q_1^{rs}(t)}, \tag{12}$$

where q_1^{rs} is the hourly demand of equipped travelers between OD pair rs. The user benefit on OD pair rs (ub^{rs}) is defined as:

$$\mathrm{ub}^{rs} = \mathrm{TS}^{rs} - C_N, \tag{13}$$

where C_N, TS^{rs}, respectively, are the service charge and the average travel time saving on OD pair rs:

$$\text{Static model}: \quad \mathrm{TS}^{rs} = B \cdot \left\{ \left(\sum_p w_{p,2}^{rs} \cdot \eta_p^{rs} \right) - \left(\sum_p w_{p,1}^{rs} \cdot \eta_p^{rs} \right) \right\}; \tag{14}$$

$$\text{Dynamic model}: \quad \mathrm{TS}^{rs} = B \cdot \left(\frac{\sum_t \sum_p f_{p,2}^{rs}(t) \cdot \eta_p^{rs}(t)}{\sum_t q_2^{rs}(t)} - \frac{\sum_t \sum_p f_{p,1}^{rs}(t) \cdot \eta_p^{rs}(t)}{\sum_t q_1^{rs}(t)} \right), \tag{15}$$

where $f_{p,i}^{rs}$ and $w_{p,i}^{rs}$ are respectively the flow and proportion of Class i travelers on route p between OD pair rs; q_i^{rs} is the hourly demand of Class i travelers between OD pair rs; η_p^{rs} denotes the average route travel time for travelers taking route p between OD pair rs. In (15), the first term in the bracket represents the average travel time of the unequipped travelers for the study horizon whereas the second term represents the average travel time of the unequipped travelers. Their difference therefore measures the average travel time savings between these two groups for the study horizon.

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[1] In defining these measures for the static and dynamic models, every precaution is taken to ensure that they are comparable and measuring the same effect.

ISP is concerned about profit. We define the profit function P to be:

$$\mathrm{P} = Q \cdot C_N - \left[\tau \cdot \theta_1 + \int_0^Q (\lambda + \mathrm{e}^{-\mu x})\,\mathrm{d}x\right]. \tag{16}$$

The first term on the right hand side is the revenue from the number of users Q, where Q is defined as:

$$\text{Static model}: \quad Q = \sum_{rs} q_1^{rs}; \tag{17}$$

$$\text{Dynamic model}: \quad Q = \sum_t \sum_{rs} q_1^{rs}(t). \tag{18}$$

The bracket in (16) represents the cost of providing the services. The first term in the bracket is the cost of collecting and processing data, which is fixed regardless of the number of users. This fixed cost models the infrastructure and operating costs to provide information quality at θ_1. τ is the proportionality parameter, interpreted as the cost of collecting and processing data per information quality. In this study, we assume a linear relationship between the fixed cost and θ_1. In general, the strategy of detector placements and therefore its cost so as to improve the quality of information is not a well-studied topic, which deserves further investigation and should form an interesting research direction. At this moment, the linear relationship assumed herein is more for convenience rather than an accurate representation. Nevertheless, this study imposes no restriction on using other functional forms, which could also be network and site dependent.

The second term in the bracket is the cost of serving Q users, which is variable. As is customary, we further assume that this variable cost per user is decreasing with the number of users. The parameter λ is the ultimate cost per user with a large user base. And μ can be interpreted as capturing the economy of scale of providing the services.

TMA is concerned with reduction in total system travel time. The total system travel time TSTT is the sum of the travel times of the equipped and unequipped travelers on each route during the study horizon, defined as:

$$\text{Static model}: \quad \mathrm{TSTT} = \sum_{rs} \sum_p \sum_i f_{p,i}^{rs} \cdot \eta_p^{rs}; \tag{19}$$

$$\text{Dynamic model}: \quad \mathrm{TSTT} = \sum_t \sum_{rs} \sum_p \sum_i f_{p,i}^{rs}(t) \cdot \eta_p^{rs}(t). \tag{20}$$

We define the relative reduction in TSTT (RT) as in (21) to capture the change in TSTT before and after the implementation of ATIS services:

$$\mathrm{RT} = \frac{\mathrm{TSTT}^{\mathrm{b}} - \mathrm{TSTT}^{\mathrm{a}}}{\mathrm{TSTT}^{\mathrm{b}}} \times 100\%. \tag{21}$$

The superscripts b, a, respectively, refer to the cases of "before" and "after" the services implementation. A positive (negative) RT refers to the case of travel time reduction (increase) due to the services.

The network market penetration of ATIS services (MP) is defined as the ratio of the number of equipped travelers over the total demand. Mathematically, it is expressed as:

$$\text{Static model:}\quad \mathrm{MP} = \frac{\sum_{rs} q_1^{rs}}{\sum_{rs} \tilde{q}^{rs}}; \tag{22}$$

$$\text{Dynamic model:}\quad \mathrm{MP} = \frac{\sum_t \sum_{rs} q_1^{rs}(t)}{\sum_t \sum_{rs} \tilde{q}^{rs}(t)}. \tag{23}$$

4. Comparison methodology

We apply both the static and dynamic mixed-equilibrium models to the same scenario. In coding the scenario, to the extent possible all the parameters are set to be identical for both models, as the objective is to compare the model outputs for the same scenario. The detailed parameters of the numerical study are provided in Section 5. In the following, we depict the procedure to determine the model results for a range of design parameters of the ATIS services, including the quality of information to be provided (θ_1) and their service charge (C_N).

(i) Start with a pair of (θ_1, C_N).
(ii) Solve the static model expressed as the VIP (10) based on the pair of (θ_1, C_N) fixed at (i).
(iii) Substitute the solution in (ii) into (11), (16), (21), (22) to determine the values of UB, P, RT, and MP.
(iv) Repeat (ii) and (iii) for other combinations of (θ_1, C_N).
(v) Plot the contours of UB, P, RT, and MP graphically for various combinations of (θ_1, C_N) on the $C_N \times \theta_1$ decision plane.
(vi) Repeat (i)–(iv) to obtain the benefit plots for the dynamic model expressed as the VIP (10).

In step (ii), given θ_1 and C_N, one can apply a number of existing algorithms to solve the VIP (10). In the past, various solution methods have been proposed for solving VIP, including projection methods (e.g. He, 1997) and Newton type methods (e.g. Taji et al., 1993). In this study, we choose the projection method developed by Han and Lo (in press). An advantage of this projection method is that it can solve VIP with co-coercive mappings, which is a less stringent requirement than the condition of strongly monotone mappings. This flexibility permits the method to solve a wider class of problems. Moreover, for our static and dynamic problems, the algorithm requires a function evaluation and a simple projection on the non-negative orthant per iteration. It does not require computationally intensive tasks such as matrix inversion, making it attractive from a computational point of view. For brevity, we do not repeat the convergence analysis of this approach but merely state the descent algorithm:

Step 1: Select positive constants ϕ and γ such that $\phi < 4\bar{\mu}$, $\gamma = \bar{\delta}(1 - \frac{\phi}{4\bar{\mu}})$, $\bar{\delta} \in (0, 2)$, $\gamma \in (0, 1)$, where $\bar{\mu}$ is the modulus constant associated with $\mathbf{F}(\mathbf{x})$, which ensures that the mapping is co-coercive: $(\mathbf{x} - \mathbf{y})^{\mathbf{T}}(\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})) \geqslant \bar{\mu}\|\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})\|^2$, $\mathbf{x}, \mathbf{y} \in \Omega$
Step 2: Start with an initial point $\mathbf{x}^0 \in \Omega$ and set $k = 0$

Step 3: Generate

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} - \gamma \cdot \mathbf{e}(\mathbf{x}^{k}, \phi)$$

where $\mathbf{e}(\mathbf{x}, \phi) = \mathbf{x} - P_{\Omega}[\mathbf{x} - \phi \cdot \mathbf{F}(\mathbf{x})]$, and P_{Ω} is the projection operator.

Step 4: Convergence check, let $\varepsilon = 0.001$ be the convergence criterion: If $\|\mathbf{e}(\mathbf{x}, \phi)\|^2 \leqslant \varepsilon$ stop, otherwise $k = k + 1$, go to Step 3

5. Numerical studies

This section aims at comparing the benefit measures of MP, UB, P and, RT obtained by the static and dynamic models. We set up two comparisons: high and low demand scenarios.

5.1. Scenario 1: High demand

For ease of results exposition and investigation, we choose a simple network, even though the formulations are applicable for general networks. The network is located in the western suburb of Hong Kong, consisting of 4 nodes, 4 links, and 2 OD pairs, as shown in Fig. 1. The two OD pairs are from node 1 to node 3 and from node 4 to node 3. OD pair (1, 3) has two routes (Routes 1 and 2) while OD pair (4, 3) has only one route (Route 3). We consider the morning peak.

As the static and dynamic models use different methods to determine route travel times, the sets of parameters used are not completely identical. The dynamic model uses more detailed network parameters such as jam density, shockwave speed, etc. Nevertheless, all the parameters that are shared by both models are set to be identical, including:

(i) Route choice and ATIS parameters: $\lambda = 0.5$, $\mu = 10$, $B =$ HK\$0.67/min, $\tau = 2500$, $\theta_2 = 0.05$ min^{-1}, $\psi = 0$.
(ii) Network parameters: Link 1–14 miles, Link 2–5 miles, Link 3–4 miles, Link 4–1 mile; each link has 2 lanes; each lane has a flow capacity of 1800 v/h; free-flow speed for each link is 60 mph.
(iii) Demand parameters: the demand rates for both OD pairs (1, 3) and (4, 3) are each 3600 vph.
(iv) Study duration: 1 h.
(v) Solution algorithm parameters: $\phi = 0.05$, $\gamma = 0.5$, and $\varepsilon = 0.001$.

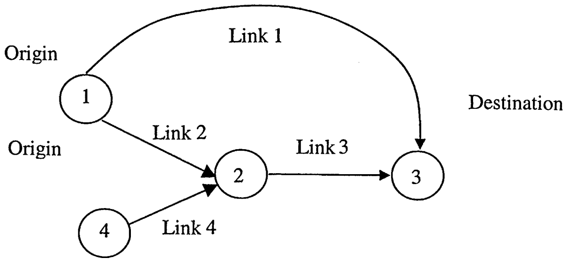


Fig. 1. The example network.

The parameters that are specific to each model include:

(i) For the static model: the practical capacity is set to be 75% of the flow capacity (Khisty and Lall, 1990, p. 500); α and β associated with the BPR function in (5b) are set to their typical values of 0.15 and 4, respectively.
(ii) For the dynamic model: jam density: 200 vehicles/mile; shockwave speed: 15 mph; time is discretized at 1-min intervals.

For each pair of ATIS design parameters (θ_1, C_N), according to the procedure in Section 4, we solve both the static and dynamic models to obtain the benefit measures. To illustrate how well the equilibrium conditions are met under the convergence tolerance of $\varepsilon = 0.001$ in the solution algorithm, the route flow proportions of the static case $(\theta_1, C_N) = (0.45, 0)$ are shown in Table 1. Columns 2 and 3 show the route flow proportions as assigned, whereas columns 4 and 5 show the proportions as determined by the corresponding logit expressions. For every route, the assigned route flow proportion and the corresponding logit expression are equal to the second decimal place, showing that the equilibrium conditions are met well under this ε value. In the interest of space, we do not report the results for the dynamic case. But the convergence tolerance of $\varepsilon = 0.001$ imposes a similar level of accuracy for the dynamic case.

Fig. 2 shows the results for OD pair (1, 3) as determined by the dynamic model when the ATIS design parameters are set at $(\theta_1, C_N) = (0.45, 0)$. The x-axis in Fig. 2 is the departure time whereas

Table 1
The route flow proportion as assigned versus as required by the logit expression

Route p	$w_{p,1}^{rs} = \frac{f_{p,1}^{rs}}{q_1^{rs}}$	$w_{p,1}^{rs} = \frac{\exp(-\theta_1 \cdot \eta_p^{rs})}{\sum_k \exp(-\theta_1 \cdot \eta_k^{rs})}$	$w_{p,2}^{rs} = \frac{f_{p,2}^{rs}}{q_2^{rs}}$	$w_{p,2}^{rs} = \frac{\exp(-\theta_2 \cdot \eta_p^{rs})}{\sum_k \exp(-\theta_2 \cdot \eta_k^{rs})}$
1	0.67	0.67	0.52	0.52
2	0.33	0.33	0.48	0.48
3	1.00	1.00	1.00	1.00

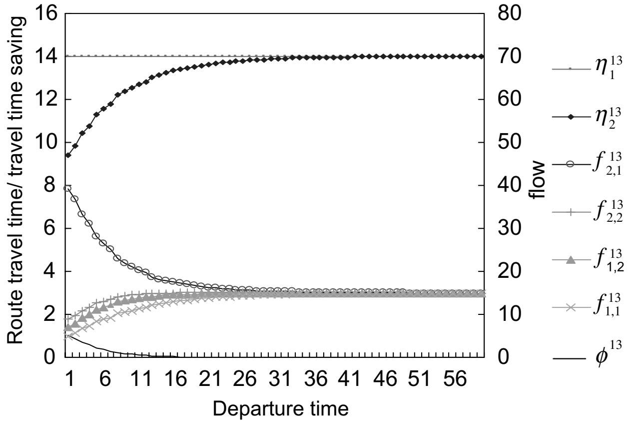
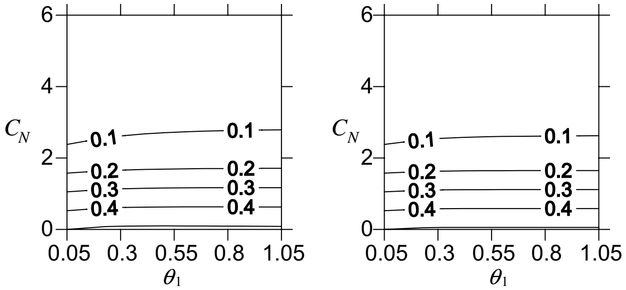


Fig. 2. Path flows, path times, and travel time savings during the departure time period for OD pair (1, 3).

the y-axis represents the values of three types of curves as labeled in the legend: route travel times, route flows, and travel time savings. The top two lines represent the actual route travel times of the two alternate routes between OD pair (1, 3). The middle four curves depict the route flows of the equipped and unequipped travelers on each of the two alternate routes. The bottom curve shows the travel time difference (savings) between the two alternate routes. Fig. 2 illustrates that route travel times, route flows, and travel time savings (and thus market penetration) all vary with the departure time, which cannot be modeled with the static model.

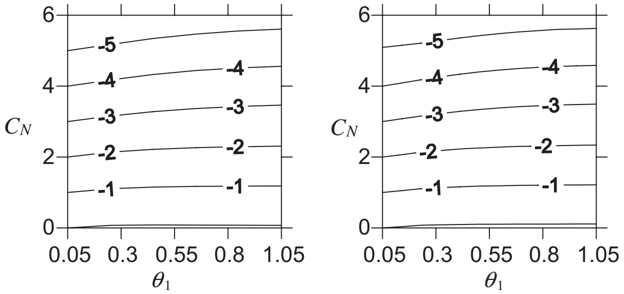
The top curve shows that the travel time on Route 1 (or Link 1) is constant and equal to its free-flow travel time of 14 min for the entire study period. For Route 2, its travel time (labeled as η_2^{13} in Fig. 2), initially at 9 min, gradually increases due to the congestion caused by earlier departures at the junction Node 2. A large proportion of equipped travelers, who depart early, take advantage of the lower travel time on Route 2. As the travel time on Route 2 gradually increases, the number of equipped travelers on Route 2 drops accordingly, whereas that on Route 1 increases slightly. ATIS services do help travelers select the better route. On the other hand, the unequipped travelers, not knowing the network condition well, split themselves between the two routes quite evenly for all departure times. The curve on travel time saving shows that initially there is a positive saving between the two routes, which gradually diminishes as earlier departures jam the merge junction at Node 2. For this reason, the market penetration or the number of equipped travelers drops over the study horizon.

In general, by following the procedure in Section 4, we obtain the four performance measures over the decision plane (θ_1, C_N) of ATIS service design. The performance measures of MP for the static and dynamic models are shown in Fig. 3(a) and (b). For a fixed value of θ_1, higher ATIS service fees lead to lower MP. Comparing Fig. 3(a) and (b), it appears that the results of the static and dynamic models are consistent. Fig. 4(a) and (b), respectively, show that the performance measures of UB for the static and dynamic models. The results shown in these figures match with those shown in Fig. 3(a) and (b). In the studied scenario, for most combinations of (θ_1, C_N), users face negative benefits. Only when the service is free would users experience a positive benefit. This is due to the small travel time saving associated with this given network and demand pattern. Again, the results of the static and dynamic models are comparable and consistent. For the benefit measure of profit, once again, Fig. 5(a) and (b) show that both models predict similar profits across a broad range of (θ_1, C_N). There appears to be an optimal combination of $(\theta_1, C_N) = (0.05, 2.1)$ whereby the ISP can maximize its profit—when the ATIS' information quality is low



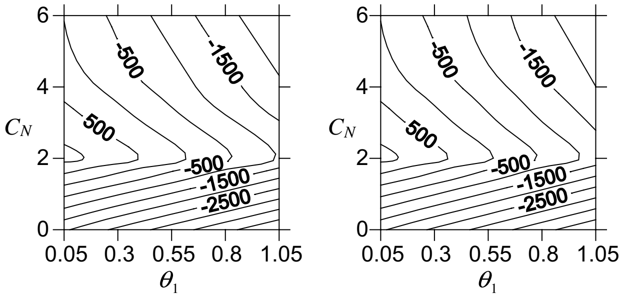
(a) static case (b) dynamic case

Fig. 3. Market penetration of ATIS services.



(a) static case (b) dynamic case

Fig. 4. Average user benefit of ATIS services.



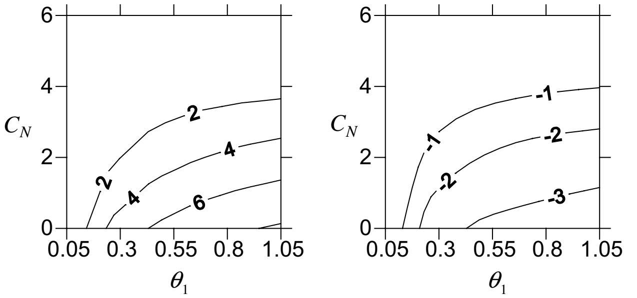
(a) static case (b) dynamic case

Fig. 5. Profit of ATIS services.

(and hence lowering the ISP's information acquisition and processing costs) and the service fee is set at 2.1 units. Both the static and dynamic models predict this same optimal combination.

These similar patterns in MP and UB predicted by both the static and dynamic models are due to the result that both models produce a low level of travel time savings for this network. In that sense, the two models are consistent. Working through the relationships expressed in (2) and (3), when the travel time savings predicted by both models are small, user benefits are largely dictated by the service charge, which is the same for both models. Therefore, both models produce similar user benefit UB patterns. Moreover, as market penetration MP and profit P are each related monotonically to UB, both models produce similar results in MP and P as well.

Finally, we examine the result of reduction in total system travel time RT. Fig. 6(a) and (b) show the measures of RT obtained by the static and dynamic models. Contrary to the earlier results, the contour pattern of Fig. 6(a) is entirely different from that of Fig. 6(b). According to the static model, the system would benefit from ATIS services for a broad range of (θ_1, C_N); the dynamic model predicts just the opposite, with negative values of RT on the entire (θ_1, C_N) plane. This opposite result in RT does not contradict the earlier similar results in UB, MP and P. The earlier results are due to similarly small travel time savings predicted by both models. As long as the travel time differences between the equipped and unequipped travelers are small, then the travel time savings are small. This relationship, however, does not specify the change in total

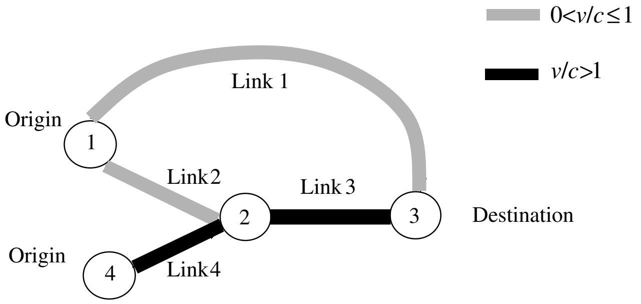


(a) static case (b) dynamic case

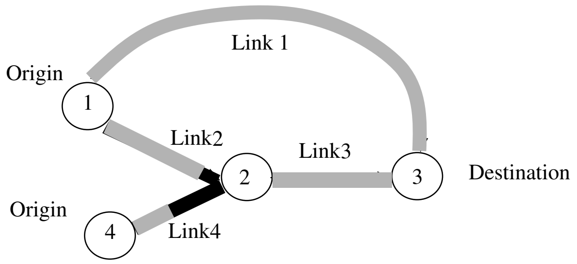
Fig. 6. Reduction in total system resulted from ATIS services in Scenario 1.

system time RT, which is of interest here. A deeper analysis shows that this diametrical result is rooted in how traffic is modeled.

To explain this phenomenon, we construct Fig. 7(a) and (b) schematically to show the network condition *before* the provision of ATIS services. Fig. 7(a) and (b) shows the results according to the static (dynamic) model. Darker shade refers to higher congestion (volume/capacity ratio greater than 1). In the static model, as physical queues and junction blockage are not considered, the entire traffic on OD pair (4, 3) is present simultaneously on Links 3 and 4. After ATIS services are introduced, better information quality lets the equipped drivers on OD pair (1, 3) realize the heavy congestion on Route 2 (which contains Link 3), who then switch to the longer but un-



(a) Static model



(b) Dynamic model

Fig. 7. The initial network congestion without ATIS services.

Table 2
The proportion of flow on Route 2 before and after the provision of ATIS services

	Before ATIS flow on Route 2 (%)	After ATIS flow on Route 2 (%)
Static model	46	40
Dynamic model	53	54

congested Route 1 (or Link 1), as shown in Table 2. Not only do those who switch save travel time, they also benefit the traffic on OD pair (4, 3) that must use Link 3. The end result is that the overall system travel time benefits from ATIS services. A key to the results of the static model is that the entire traffic on OD pair (4, 3) is simultaneously loaded onto Link 3. This congestion on Link 3 enters into the consideration of the equipped drivers from OD pair (1, 3), leading to some of them switch away from Route 2.

As for the results of the dynamic model, Fig. 7(b) shows the network status *before* the provision of ATIS services. The bottleneck junction at Node 2 causes two queues forming on Links 2 and 4. Link 3, downstream of Node 2, remains uncongested, contrary to the results of the static model. In contemplating about route switching, the equipped drivers from OD pair (1, 3) are only affected by the queue on Link 2; Link 3 has no congestion. Table 2 shows that the dynamic model predicts that Route 2 attracts a modest gain in traffic, as ATIS services let the equipped drivers realize its shorter travel time. Moreover, Fig. 8, which plots the traffic on Route 2 as a function of departure time, shows that ATIS services lead to a higher flow on Route 2 early on (so as to take advantage of its shorter travel time before congestion starts), as compared with the network before the provision of ATIS services. The end result is that the equipped drivers benefit slightly by switching to Route 2 early on whereas the entire traffic from OD pair (4, 3) faces a more severe jam at the bottleneck junction, which leads to an increase in the total system travel time. In this analysis, the absence of congestion on Link 3 is a differentiating factor as compared with the static model. In reality, one expects the dynamic model to provide a much better depiction of the actual traffic pattern.

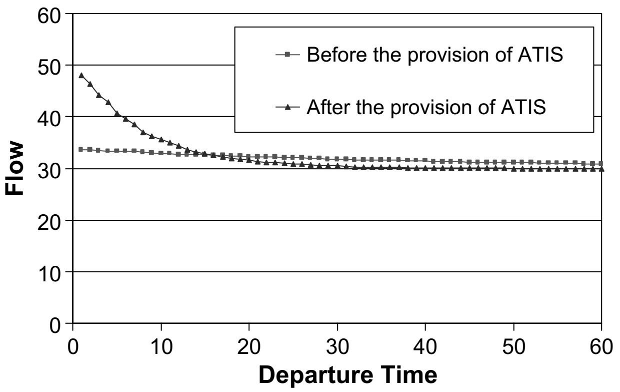
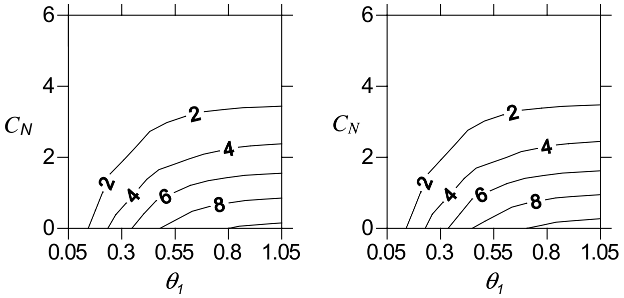


Fig. 8. The flow on Route 2 before and after the provision of ATIS services.



(a) static case (b) dynamic case

Fig. 9. Reduction in total system resulted from ATIS services in Scenario 2.

5.2. Scenario 2: Low demand

The only difference between Scenarios 1 and 2 are the demand rates. In this scenario, the demand rate of each OD pair is lowered from 3600 vph (Scenario 1 demand rates) to 720 vph. Again, in this scenario, since both models predict a low level of travel time savings for this network, they produce similar patterns in MP, UB, and P, which are not shown. Fig. 9a and b show the measures of RT obtained by the static and dynamic models in this scenario. Unlike Scenario 1, the results of the two models agree completely. Due to these low demand rates, junction blockage does not occur. Both models predict similar traffic patterns and similar system travel times. Comparing the results of Scenarios 1 and 2, it appears that the occurrence of junction blockage or not causes the different predictions between the static and dynamics models.

These comparison results are obviously case specific and network dependent. Nevertheless, they illustrate that even if some aspects of the results from these two types of models are similar, other aspects can be directly opposite. Based on this finding, it appears that traffic dynamics is too important to be simplified to a simple link performance function, especially for congested networks wherein junction blockage is common. The results, however, illustrate that under low demand conditions, both paradigms could produce virtually identical trends in all aspects. Nevertheless, it is difficult to determine a priori how low is low enough so that junction blockage does not occur. At the end, it is hard to be assured of the results if one is not equipped with a dynamic model that takes into account more detailed traffic dynamics.

6. Concluding remarks

This paper developed a cell-based variational inequality model for the DTA problem to evaluate the impact of ATIS services. It considers two classes of drivers: equipped and unequipped. Both classes are modeled to follow the SDUO conditions, with the equipped drivers having a lower perception variation of travel time due to the availability of better information. As inherited from the modeling platform developed in Lo and Szeto (2002b), this model represents traffic dynamics as according to the Cell Transmission Model, including such physical effects as queue spillback, shockwaves, etc.

One main objective of this study is to compare and contrast the static versus the dynamic modeling paradigms in evaluating the impacts of ATIS services, including user benefits, profit, and reduction in total system travel time. We set up two scenarios to evaluate the two modeling paradigms while carefully controlling the scenario parameters so as to ensure that the results between the two models are comparable. One would expect the two paradigms not to produce identical numerical results. However, it is not obvious whether they would agree in terms of identifying trends. The results show that the benefits estimated by these two models could be very different. Even if some aspects of the results are very similar, other aspects can be directly opposite. The discrepancy is mainly attributed to the fundamental characteristics of how traffic is modeled. It makes a big difference whether traffic dynamic is modeled indirectly via a link performance function, or is represented realistically with physical queues. The numerical studies seem to indicate that simplifications from the physical queue representation are inadequate in producing correct results.

Acknowledgements

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Appendix A

This Appendix A briefly describes the basic principles of CTM and the method to determine route travel time through CTM for any given set of route flows. In the following, we distinguish the difference between two time indices: t refers to the traffic's departure time; whereas ω is used to keep track of the departed traffic along its course in the network.

A.1. Cell-transmission model (CTM): basic principles

The Cell Transmission Model (Daganzo, 1994, 1995) is a convergent numerical approximation to the hydrodynamic or kinematic wave model of traffic flow that covers the full range of the fundamental diagram. Numerical studies in Lo and Szeto (2002c) show that CTM can capture realistic traffic dynamics such as shock waves, queue formation, and queue dissipation, and dynamic traffic interactions across multiple links such as queue spillbacks.

By discretizing the road into homogenous sections (or cells) and time into intervals such that the cell length is equal to the distance traveled by free-flowing traffic in one time interval, then the Lighthill and Whitham (1955) and Richards (1956) (LWR) model is approximated by this set of recursive equations:

$$n_j(\omega+1) = n_j(\omega) + y_j(\omega) - y_{j+1}(\omega); \tag{A.1}$$

$$y_j(\omega) = \min\left\{n_{j-1}(\omega), Q_j(\omega), (W/V)[N_j(\omega) - n_j(\omega)]\right\}, \tag{A.2}$$

where the subscript j refers to cell j, and $j+1$ ($j-1$) represents the cell downstream (upstream) of j. The variables $n_j(\omega)$, $y_j(\omega)$, $N_j(\omega)$ denote the number of vehicles, the actual inflow, and the maximum number of vehicles (or holding capacity) that can be held in cell j at time ω, respectively. The variables $Q_j(\omega)$, V, W denote, respectively, the saturation flow, free-flow speed, and shockwave speed. The key is to determine $y_j(\omega)$ from the minimization (A.2). Once this is accomplished, $n_j(\omega)$ can be determined recursively from the linear equation (A.1). Eqs. (A.1) and (A.2) provide the basic principle of modeling traffic flow on a series of straight cells.

To apply this principle to a general network with multiple OD pairs, three extensions are required: (a) modeling merge and diverge junctions, (b) differentiating the OD specific traffic, and (c) maintaining the first-in-first-out (FIFO) property. The detailed mathematical operations to ensure these conditions are addressed in Daganzo (1995). In brevity, given a set of time-sequenced inflow $f_p^{rs}(t)$, based on the traffic propagation conditions (such as the recursive equations (A.1) and (A.2)), one can obtain a set of unique occupancy counts $n_{j,p}(\omega)$ for traffic in cell j on route p at any time instance ω.

A.2. Determination of actual route travel time

Knowing the occupancy of each package of traffic on route p in origin cell r and destination cell s at each time instance, the actual en route travel time $\eta_p^{rs}(t)$ of flow $f_p^{rs}(t)$ can be determined through the use of cumulative counts. Let $\lambda_p^r(t)$ be the cumulative traffic departing from cell r on route p at time t and $\lambda_p^s(\omega)$ be the cumulative traffic arriving at cell s on route p at time ω, defined by:

$$\lambda_p^r(t) = \sum_{t' \leqslant t} n_{r,p}(t'); \tag{A.3}$$

$$\lambda_p^s(\omega) = \sum_{\omega' \leqslant \omega} n_{s,p}(\omega'). \tag{A.4}$$

If time is discretized, subject to the en route conditions, there is no guarantee that the entire packet $f_p^{rs}(t)$ will arrive at the destination s in the same discretized time tick ω. To ensure that the entire departing traffic $f_p^{rs}(t)$ has one uniquely determined average en route travel time $\eta_p^{rs}(t)$, the following averaging scheme is adopted. Mathematically, this scheme can be stated as:

$$\eta_p^{rs}(t) = \frac{\int_{\lambda_p^r(t-1)}^{\lambda_p^r(t)} \left[\lambda_p^{s^{-1}}(\upsilon) - \lambda_p^{r^{-1}}(\upsilon)\right] \mathrm{d}\upsilon}{f_p^{rs}(t)}. \tag{A.5}$$

Essentially, this scheme finds the average travel time of each departing traffic packet by summing and then averaging the actual travel times of its sub-packets.

Summarizing, as CTM determines a unique set of cumulative counts from route flows and the averaging scheme determines a unique set of route travel times from the cumulative count curves, through these two processes, one establishes a one-to-one mapping from route flows to route travel times.

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